Lecture 13

Reading Material

Reading:

Chapter 8 Solymar and Walsh

N and P-Type Semiconductors

- Using the equations we developed previously for carrier density assuming Maxwell-Boltzman distribution for the tails of the Fermi-Dirac statistics and referencing the valance band energy to 0eV and the conduction band energy to E_g we can write

$$\text{N - Type}\left\{N_e \approx N_c \exp\left(-\frac{E_g - E_F}{\kappa_B T}\right) \approx N_D^+ \approx N_D\left[1 - \exp\left(1 + \frac{E_F - E_D}{\kappa_B T}\right)^{-1}\right]\right.$$

$$\text{P - Type}\left\{N_h \approx N_v\left[1 - \exp\left(-\frac{E_g - E_F}{\kappa_B T}\right)\right] \approx N_A^- \approx N_A \exp\left(1 + \frac{E_A - E_F}{\kappa_B T}\right)^{-1}\right.$$

- Lest first assume that $\frac{E_F - E_D}{\kappa_B T} \Rightarrow -\infty$

$$N_e \approx N_c \exp\left(-\frac{E_g - E_F}{\kappa_B T}\right) \approx N_D^+ \approx N_D$$

$$N_c e^{-\frac{E_g}{\kappa_B T}} e^{\frac{E_F}{\kappa_B T}} = N_D$$

$$C e^{\frac{E_F}{\kappa_B T}} = N_D, \text{ where } C = N_c e^{-\frac{E_g}{\kappa_B T}}$$

Note linear dependence on T and Ln dependence on N_D. N_D has small effect in this regime.

$$E_F = \kappa_B T \ln\left(\frac{N_D}{C}\right)$$

Temperature Dependence of E_F

$$\text{recall that } N_c \exp\left(-\frac{E_g - E_F}{\kappa_B T}\right) \approx N_D\left(1 + \exp\left(-\frac{E_F - E_D}{\kappa_B T}\right)\right)$$

$$\text{let } x = e^{\frac{E_F}{\kappa_B T}}$$

$$\text{then } N_c \exp\left(-\frac{E_g}{\kappa_B T}\right) x = N_D + N_D \exp\left(\frac{E_D}{\kappa_B T}\right) x^{-1}$$

$$N_D \exp\left(\frac{E_D}{\kappa_B T}\right) + N_D x - N_c \exp\left(-\frac{E_g}{\kappa_B T}\right) x^2 = 0$$

$$A + Bx + Cx^2 = 0$$

$$A = N_D \exp\left(\frac{E_D}{\kappa_B T}\right);\ B = N_D;\ \mathrm{C} = -N_c \exp\left(-\frac{E_g}{\kappa_B T}\right)$$

$$D = B^2 - 4AC = N_D^2 + 4N_c N_D \exp\left(-\frac{E_g - E_D}{\kappa_B T}\right) > 0, \text{ so two real roots}$$

Temperature Dependence of E_F

$$x_1 = \frac{-B+\sqrt{D}}{2A} = \frac{-N_D + \sqrt{N_D{}^2 - 4N_c N_D \exp\left(-\frac{E_g - E_D}{\kappa_B T}\right)}}{2N_D \exp\left(\frac{E_D}{\kappa_B T}\right)} = \exp\left(\frac{E_F}{\kappa_B T}\right)$$

$$x_2 = \frac{-B-\sqrt{D}}{2A} = \frac{-N_D - \sqrt{N_D{}^2 - 4N_c N_D \exp\left(-\frac{E_g - E_D}{\kappa_B T}\right)}}{2N_D \exp\left(\frac{E_D}{\kappa_B T}\right)} = \exp\left(\frac{E_F}{\kappa_B T}\right)$$

$$2\exp\left(\frac{E_F - E_D}{\kappa_B T}\right) = -1 + \sqrt{1 - 4\frac{N_c}{N_D}\exp\left(-\frac{E_g - E_D}{\kappa_B T}\right)}$$

$$2\exp\left(\frac{E_F - E_D}{\kappa_B T}\right) = -\left(1 + \sqrt{1 - 4\frac{N_c}{N_D}\exp\left(-\frac{E_g - E_D}{\kappa_B T}\right)}\right)$$

Temperature Dependence of E_F

Assuming $E_F > E_D$ at very low temperatures (only donors contribute to free carriers)

$$2\exp\left(\frac{E_F - E_D}{\kappa_B T}\right) = -\left(1 + \sqrt{1 - 4\frac{N_c}{N_D}\exp\left(-\frac{E_g - E_D}{\kappa_B T}\right)}\right)$$

Using $\sqrt{1+x} \approx 1 + \frac{1}{2}x$ for $\|x| < 1|$

$$\exp\left(\frac{E_F - E_D}{\kappa_B T}\right) = -1 + \frac{N_c}{N_D}\exp\left(-\frac{E_g - E_D}{\kappa_B T}\right)$$

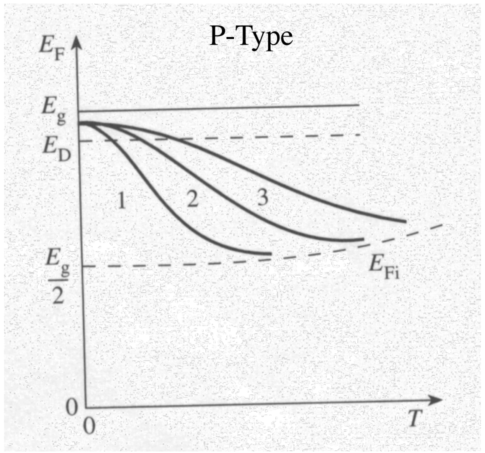
$$\frac{E_F - E_D}{\kappa_B T} = \frac{E_D - E_g}{\kappa_B T} + \ln\left(\frac{N_c}{N_D}\right)$$

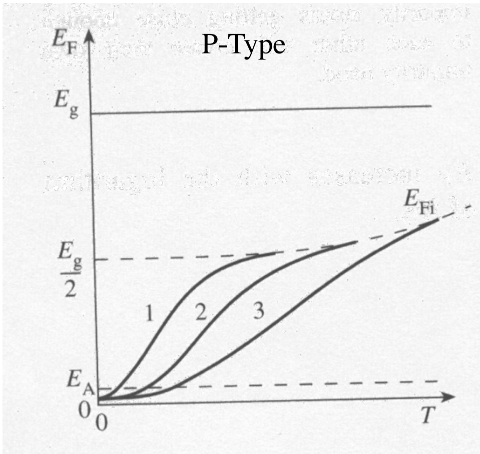
$$E_F - E_D = E_D - E_g + \kappa_B T \ln\left(\frac{N_c}{N_D}\right)$$

$$E_F \approx \left.\frac{E_D - E_g}{2}\right|_{T \to 0}$$

At very high temperatures, most of the covalent bonds can be broken and the valence band electrons can dominate the number of free carriers so

$$E_F \approx \left.\frac{E_g}{2}\right|_{T \to High}$$





Semiconductor Conductivity-Non-Ionized Impurities

- The conductivity can be defined by the effective mass (m*) and the mean free time between collisions (τ)

$$\sigma = \frac{q^2}{m^*}\tau N_e$$

- The mean free path between collisions can be written as $l = \tau \upsilon_g \approx \tau T^{1/2}$ where T, the temperature is responsible for molecular or atomic vibration that causes the mean free velocity to change
- Now lets assume that the mean free path between collisions is inversely proportional to the probability that an electron will scatter off a vibrating atom or molecule, or $l \propto T^{-1}$, then we can write the mean free time between collisions due to thermal vibrations as

$$\tau_{thermal} = \frac{l}{\upsilon_g} \approx \frac{1}{T \bullet T^{1/2}} = \frac{1}{T^{3/2}}$$

Semiconductor Conductivity-Ionized Impurities

- For ionized impurities, the positive charge of donors or negative chare of acceptors will impact the mean velocity of the carriers. If this electrostatic energy (given by some Zq^2) is of the same magnitude as the thermal vibration (thermal energy) we can write

$$\underbrace{\frac{Zq^2}{4\pi\varepsilon r_s}}_{\text{Electrostatic Energy}} = \underbrace{\frac{3}{2}\kappa_B T}_{\text{Thermal Energy}}$$

- Solving for the radius r_s

$$r_s = \frac{Zq^2}{6\pi\varepsilon\kappa_B T}$$

- The ion cross scattering section is related to the radius by

$$S_s = \pi r_S^2 = \frac{1}{\pi}\left(\frac{Zq^2}{6\varepsilon\kappa_B T}\right)^2$$

- Assuming that the mean free path is inversely proportional to the cross scattering section

$$\tau_{\text{ionizing impurity}} = \frac{l}{T^{1/2}} \approx \frac{1}{S_S}\frac{l}{T^{1/2}} \propto \frac{1}{T^{3/2}}$$

Scattering due to combined non-ionized and ionized impurities

- We can calculate and effective mean scattering time by considering both contributions as follows

$$\frac{1}{\tau_{\text{eff}}} = \frac{1}{\tau_{\text{thermal}}} + \frac{1}{\tau_{\text{thermal}}} \frac{1}{T^{3/2}}$$

$$\tau_{thermal} = \frac{l}{\upsilon_g} \approx \frac{1}{T \bullet T^{1/2}} = \frac{1}{T^{3/2}}$$

$$\tau_{\text{ionizing impurity}} = \frac{l}{T^{1/2}} \approx \frac{1}{S_S} \frac{l}{T^{1/2}} \propto \frac{1}{T^{3/2}}$$